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A TIME-VARIANT FUZZY-STOCHASTIC MARKOV MODEL FOR NON-HOMOGENEOUS WORKFORCE SYSTEMS UNDER EPISTEMIC AND ALEATORY UNCERTAINTY

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ABSTRACT

Modern workforce systems operate in highly dynamic environments characterized by structural evolution, uncertain transition mechanisms, and incomplete managerial information. Traditional homogeneous stochastic workforce models often fail to capture real organizational complexities arising from time-dependent policies, ambiguous human behavior, and fluctuating operational conditions. This study proposes a Time-Variant Fuzzy–Stochastic Markov Model for analyzing and optimizing non-homogeneous workforce systems under combined epistemic and aleatory uncertainty. The framework integrates stochastic Markov transition dynamics with fuzzy set theory to simultaneously model randomness inherent in workforce mobility and imprecision arising from subjective assessments, policy ambiguity, and incomplete data. The model introduces time-dependent transition probability structures that accommodate evolving organizational policies, promotion rules, recruitment strategies, and attrition patterns. Aleatory uncertainty is represented through probabilistic state transitions, while epistemic uncertainty is incorporated via fuzzy membership functions describing uncertain managerial judgments and workforce performance evaluations. A hybrid fuzzy–stochastic formulation is developed to estimate workforce distributions across grades over multiple planning horizons. Stability conditions, equilibrium behavior, and system adaptability are analytically examined, demonstrating improved predictive capability compared with classical homogeneous Markov approaches. Simulation experiments illustrate how the proposed framework supports strategic manpower planning, risk-aware decision-making, and adaptive workforce control under uncertain environments. Results show enhanced robustness in forecasting staffing levels, minimizing skill imbalance, and maintaining organizational sustainability despite time-varying disruptions. The proposed model provides a unified analytical foundation for modern workforce analytics where uncertainty arises from both randomness and knowledge limitations. This study contributes a scalable mathematical framework applicable to public institutions, healthcare systems, manufacturing organizations, and technology-driven enterprises seeking resilient workforce optimization under complex uncertainty structures.

Keywords: Time-Variant Markov Model; Fuzzy–Stochastic Systems; Workforce Mobility Optimization; Epistemic and Aleatory Uncertainty.

Introduction

Workforce planning and manpower system analysis have evolved significantly from early administrative record-keeping practices to sophisticated analytical and data-driven decision frameworks. Historically, workforce management in pre-industrial societies relied primarily on informal apprenticeship structures and rigid hierarchical labor allocation governed by social norms rather than scientific optimization. During the Industrial Revolution, rapid expansion of manufacturing organizations introduced the first systematic attempts to manage labor productivity, workforce allocation, and operational efficiency. Early management theorists such as Frederick W. Taylor emphasized scientific management principles, advocating measurement, standardization, and optimization of human labor processes. While these approaches improved productivity, they largely ignored uncertainty, workforce mobility, and human behavioral variability.

The emergence of operations research during and after World War II marked a turning point in workforce modeling. Mathematical optimization, probability theory, and stochastic processes began to inform personnel planning decisions. Early manpower models treated organizations as deterministic systems with fixed transition structures, assuming stable promotion policies and predictable workforce behavior (Bartholomew, 1967). However, real organizational environments soon proved far more dynamic than these assumptions allowed. Employee mobility, retirement patterns, recruitment fluctuations, and policy reforms introduced randomness that deterministic models could not adequately capture.

The introduction of stochastic modeling frameworks, particularly Markov chains, revolutionized workforce analytics. Foundational work by Andrey Markov established the mathematical basis for modeling transitions between system states using probabilistic dependence. Markov manpower models subsequently became widely adopted for forecasting workforce distributions, analyzing promotion structures, and predicting long-term staffing equilibria (Vassiliou & Georgiou, 2009). These models enabled organizations to represent workforce grades as states and employee movements as transition probabilities, thereby transforming manpower planning into a mathematically tractable problem.

Despite their analytical elegance, classical Markov workforce models relied heavily on the assumption of homogeneity and time-invariant transition probabilities. In practice, modern organizations operate within rapidly changing environments shaped by globalization, technological disruption, demographic shifts, and policy reforms. Workforce transitions rarely remain stationary; promotion rates, recruitment strategies, and attrition mechanisms evolve over time due to economic shocks, institutional restructuring, and managerial interventions (De Feyter, Guerry, & Vassiliou, 2011). Consequently, non-homogeneous and time-variant modeling approaches emerged to better reflect organizational reality.

Parallel to developments in stochastic modeling, researchers recognized that uncertainty in workforce systems arises not only from randomness but also from incomplete knowledge. Classical probability theory captures aleatory uncertainty variability inherent in random events but fails to address epistemic uncertainty originating from vague managerial judgments, subjective performance evaluation, and insufficient data. The pioneering work of Lotfi A. Zadeh introduced fuzzy set theory as a mathematical tool for representing imprecision and linguistic ambiguity (Zadeh, 1965). Fuzzy modeling enabled analysts to formalize qualitative concepts such as employee competence, promotion readiness, or organizational flexibility that cannot be precisely quantified.

The integration of fuzzy systems with stochastic processes has therefore become a major research frontier in workforce analytics. Hybrid fuzzy–stochastic models attempt to unify probabilistic randomness and informational vagueness within a single analytical framework. Such integration is particularly relevant in contemporary workforce systems characterized by digital transformation, hybrid work arrangements, evolving skill requirements, and policy-driven structural changes. Organizations increasingly require predictive models capable of adapting to both uncertain events and incomplete managerial information.

Recent studies have emphasized non-homogeneous workforce systems where transition probabilities vary across time horizons, organizational stages, or external economic conditions (Georgiou & Tsantas, 2018). Time-variant approaches allow manpower models to incorporate dynamic recruitment policies, evolving retirement regulations, and fluctuating labor market conditions. Nevertheless, existing frameworks often treat stochastic and epistemic uncertainties separately, limiting their ability to capture the full complexity of real workforce environments.

Modern workforce management further faces unprecedented challenges arising from automation, artificial intelligence adoption, global competition, and workforce diversification. Decision-makers must simultaneously manage skill shortages, succession planning risks, and organizational resilience under uncertainty. Traditional models that assume perfect information or static structures are increasingly inadequate for strategic planning. There is therefore a growing need for analytical frameworks capable of modeling workforce evolution as a dynamic, adaptive, and uncertainty-driven system.

Motivated by these limitations, this study develops a **Time-Variant Fuzzy–Stochastic Markov Model** for non-homogeneous workforce systems operating under combined epistemic and aleatory uncertainty. The proposed framework extends classical manpower Markov models by introducing time-dependent transition structures while embedding fuzzy membership representations of managerial uncertainty. By integrating probabilistic state dynamics with fuzzy inference mechanisms, the model captures both stochastic variability and knowledge imprecision within a unified mathematical formulation.

The contribution of this work is threefold. First, it generalizes traditional homogeneous Markov manpower systems into a flexible time-variant structure reflecting realistic organizational evolution. Second, it incorporates fuzzy representations of epistemic uncertainty, enabling decision models to account for subjective assessments and incomplete data. Third, it provides an analytical foundation for risk-aware workforce planning capable of supporting adaptive policy decisions in uncertain environments. Ultimately, this research positions workforce systems as complex adaptive processes rather than static administrative structures. By bridging stochastic modeling and fuzzy uncertainty theory, the proposed framework advances modern manpower analytics toward resilient, data-informed, and policy-responsive workforce optimization suitable for contemporary organizational challenges.

Literature Review

Workforce system modeling has undergone continuous intellectual evolution driven by advances in probability theory, optimization science, artificial intelligence, and decision analytics. The development of manpower planning models reflects a broader transition from deterministic administrative management toward uncertainty-aware computational frameworks capable of addressing dynamic organizational realities. This section critically reviews the theoretical and methodological foundations underlying workforce modeling, focusing on stochastic manpower systems, non-homogeneous Markov structures, fuzzy uncertainty modeling, and hybrid fuzzy–stochastic approaches.

Early Foundations of Workforce Modeling

The earliest analytical approaches to manpower planning emerged alongside industrial expansion in the early twentieth century. Management theorists, notably Frederick W. Taylor, introduced scientific management principles emphasizing productivity measurement, task specialization, and efficiency optimization. Although these frameworks established systematic workforce administration, they treated human resources as deterministic production inputs and neglected uncertainty inherent in workforce dynamics.

Subsequent developments in operations research during the mid-twentieth century introduced mathematical rigor into organizational planning. Workforce allocation problems began to be modeled using linear programming, queueing theory, and demographic transition analysis. A major breakthrough occurred with the introduction of stochastic manpower models, where workforce evolution was represented probabilistically rather than deterministically. Bartholomew (1967) pioneered stochastic modeling of social and organizational processes, demonstrating that personnel movements could be analyzed using transition probability structures analogous to population dynamics. These early models assumed stationary environments and stable transition mechanisms. However, real organizations quickly demonstrated variability arising from economic cycles, technological change, and policy interventions, exposing limitations of static manpower models.

Markov Chain Models in Workforce Systems

The mathematical theory of stochastic state transitions originated from the work of Andrey Markov, whose formulation of dependent probabilistic sequences established the foundation of modern Markov processes. Markov chain models became highly attractive for workforce planning because organizational hierarchies naturally resemble state-transition systems, where employees move between grades through promotion, transfer, or exit. Markov manpower models represent workforce grades as discrete states and employee mobility as transition probabilities. These models allow analysts to forecast long-term workforce distributions, evaluate promotion policies, and determine steady-state staffing levels (Vassiliou & Georgiou, 2009). Applications expanded across government agencies, healthcare institutions, academic systems, and industrial organizations.

Despite their widespread adoption, classical Markov manpower models depend heavily on the assumption of homogeneity that transition probabilities remain constant over time. Such an assumption rarely holds in practice. Workforce systems are influenced by regulatory reforms, demographic changes, economic instability, and organizational restructuring. Consequently, homogeneous Markov models often fail to capture structural evolution and policy responsiveness. Researchers therefore introduced non-homogeneous Markov models in which transition matrices vary over time. Studies demonstrated that time-dependent transition structures significantly improve forecasting accuracy and managerial relevance (De Feyter et al., 2011). Non-homogeneous models allow workforce planners to incorporate changing recruitment rates, evolving promotion criteria, and variable attrition dynamics, thereby aligning mathematical models with real organizational behavior.

Limitations of Purely Stochastic Workforce Models

While stochastic models successfully capture randomness, they remain limited when uncertainty arises from incomplete knowledge rather than probabilistic variability. Workforce decisions frequently depend on qualitative assessments such as managerial judgment, employee potential, organizational culture, and policy ambiguity. These factors cannot always be expressed using precise probability distributions. Traditional stochastic frameworks assume that transition probabilities are known or reliably estimated from historical data. In reality, workforce data are often sparse, subjective, or inconsistent. For example, performance evaluation processes involve linguistic descriptors such as “excellent,” “adequate,” or “developing,” which cannot be precisely quantified using classical probability theory.

This limitation highlights the distinction between **aleatory uncertainty**, arising from inherent randomness, and **epistemic uncertainty**, resulting from incomplete or imprecise information. Modern workforce analytics increasingly requires modeling approaches capable of addressing both uncertainty types simultaneously.

Fuzzy Set Theory and Epistemic Uncertainty

A major advancement in uncertainty modeling emerged with the introduction of fuzzy set theory by Lotfi A. Zadeh. Fuzzy logic provides mathematical mechanisms for representing vagueness through membership functions rather than binary classifications (Zadeh, 1965). Unlike classical sets, where elements either belong or do not belong to a category, fuzzy sets allow partial membership degrees between zero and one. Fuzzy modeling proved particularly effective in human-centered systems where precise numerical measurement is difficult. Applications rapidly expanded into decision science, control engineering, medical diagnosis, and management systems. Within workforce analytics, fuzzy approaches enable representation of uncertain promotion readiness, ambiguous skill competency, and subjective managerial evaluations.

Researchers began applying fuzzy models to manpower planning problems to incorporate expert knowledge and qualitative judgment into analytical frameworks. Fuzzy manpower systems allow decision-makers to evaluate workforce performance using linguistic variables while maintaining mathematical tractability (Zimmermann, 2001). However, purely fuzzy models lack mechanisms for capturing stochastic randomness inherent in workforce mobility.

Hybrid Fuzzy–Stochastic Workforce Modeling

Recognizing the complementary strengths of stochastic and fuzzy theories, recent research has focused on hybrid fuzzy–stochastic modeling frameworks. These approaches integrate probabilistic transitions representing aleatory uncertainty with fuzzy representations of epistemic uncertainty.

Hybrid models offer several advantages:

1. Simultaneous handling of randomness and vagueness,
2. Improved robustness under incomplete data,
3. Enhanced decision support for strategic planning, and
4. Realistic representation of human behavioral variability.

In workforce systems, fuzzy–stochastic integration enables transition probabilities to depend not only on historical statistics but also on uncertain managerial perceptions and policy flexibility. Studies have demonstrated that hybrid models outperform classical stochastic frameworks in forecasting workforce imbalance and evaluating policy risk under uncertain conditions (Dubois & Prade, 1997).

Nevertheless, many existing hybrid manpower models remain limited by static assumptions or simplified uncertainty interactions. Most studies do not adequately incorporate time-variant structures, leaving a gap between theoretical models and dynamically evolving organizational environments.

Time-Variant and Non-Homogeneous Workforce Systems

Modern organizations operate in environments characterized by continuous transformation driven by globalization, automation, digitalization, and shifting labor markets. Workforce systems therefore exhibit strong non-stationary behavior. Promotion opportunities, recruitment strategies, training investments, and retirement policies change over time, requiring dynamic modeling frameworks. Time-variant manpower models extend traditional Markov systems by allowing transition probabilities to evolve across planning horizons. Such models capture organizational learning, policy adaptation, and environmental shocks. Georgiou and Tsantas (2018) emphasized that non-homogeneous manpower systems better reflect real workforce evolution, particularly in public-sector institutions and technology-driven enterprises.

However, time-variant stochastic models alone still assume precise knowledge of evolving transition structures. In practice, future policy effects and workforce responses are uncertain and partially unknown. Integrating fuzzy epistemic modeling into time-dependent stochastic systems therefore represents a natural progression in manpower analytics research.

Workforce Analytics in the Era of Intelligent Organizations

The emergence of intelligent organizations and data-driven management has transformed workforce planning into a strategic analytics problem. Artificial intelligence, digital workforce monitoring, and predictive analytics generate large volumes of data but simultaneously introduce new uncertainty sources related to model assumptions, algorithm bias, and incomplete information.

Modern workforce systems must address:

- skill obsolescence due to automation,
- hybrid and remote work arrangements,
- demographic transitions,
- organizational resilience under disruption, and
- policy-driven employment restructuring.

These challenges require adaptive models capable of learning, updating, and responding to evolving uncertainty conditions. Recent studies increasingly advocate integrated analytical frameworks combining stochastic modeling, fuzzy reasoning, and dynamic system analysis to support resilient workforce optimization.

Research Gap and Motivation

A critical synthesis of existing literature reveals several unresolved challenges:

1. Classical manpower models assume homogeneity and stationarity.
2. Pure stochastic models ignore epistemic uncertainty arising from incomplete knowledge.
3. Pure fuzzy models lack rigorous representation of random workforce mobility.
4. Many hybrid approaches fail to incorporate time-dependent organizational evolution.

Consequently, there remains a need for a unified analytical framework capable of modeling non-homogeneous workforce systems under both aleatory and epistemic uncertainty within a time-variant environment.

Contribution of the Present Study

The present research addresses these limitations by proposing a Time-Variant Fuzzy–Stochastic Markov Model integrating:

- non-homogeneous Markov transition dynamics,
- fuzzy representations of managerial uncertainty,
- simultaneous modeling of aleatory and epistemic uncertainty, and
- adaptive workforce evolution across multiple planning horizons.

By bridging stochastic manpower theory with fuzzy uncertainty modeling, the proposed framework advances workforce analytics toward realistic, uncertainty-aware, and policy-responsive decision support systems suitable for contemporary organizations.

Mathematical Model Formulation

System Description

Consider an organizational workforce system consisting of **N hierarchical grades** observed over discrete time periods
$t = 0, 1, 2, \dots, T$.
Employees may move between grades through promotion, demotion, transfer, recruitment, or exit. The workforce system is assumed to be **non-homogeneous**, meaning transition mechanisms vary with time due to organizational policies, economic conditions, and managerial decisions.
Let:
$W(t) = [\, W_1(t), W_2(t), \dots, W_n(t) \,]^T$
represent the workforce state vector at time t, where
$W_i(t)$ = number of employees in grade i at time t.
The objective is to model workforce evolution under **combined stochastic randomness (aleatory uncertainty) and imprecise managerial information (epistemic uncertainty).**

Time-Variant Stochastic Transition Structure

Let
$P(t) = [\, p_{ij}(t) \,]$
be the **time-dependent transition probability matrix**, where
$p_{ij}(t)$ = probability that an employee moves from grade i to grade j during period t.
The transition matrix satisfies:
$0 \le p_{ij}(t) \le 1$
and
$\Sigma_j \, p_{ij}(t) = 1$ for all i.
Because organizational policies evolve, P(t) changes with time, making the system **non-homogeneous**.
The stochastic workforce evolution is defined as:
$W(t + 1) = P(t)\, W(t) + R(t) - L(t)$
where:
R(t) = recruitment vector at time t
L(t) = loss (attrition/retirement) vector.

Modeling Aleatory Uncertainty

Aleatory uncertainty represents randomness inherent in workforce mobility.
Assume transitions follow probabilistic behavior such that:
$E[\, W(t + 1) \mid W(t) \,] = P(t)\, W(t)$
and workforce variance evolves as:
$Var[\, W(t + 1) \,] = P(t)\, Var[\, W(t) \,]\, P(t)^T + \Sigma(t)$

where:

$\Sigma(t)$ = covariance matrix describing random workforce fluctuations.

This stochastic formulation captures unpredictable employee behavior, resignations, and random organizational shocks.

Representation of Epistemic Uncertainty Using Fuzzy Sets

In real organizations, transition probabilities are often uncertain due to incomplete data or subjective managerial judgment. To incorporate epistemic uncertainty, fuzzy set theory is introduced.

Let each transition probability be represented by a **fuzzy number:**

$\tilde{p}_{ij}(t)$

defined through a membership function:

$\mu_{ij}(x) : [0,1]$

which expresses the degree of confidence that x represents the true transition probability.

A triangular fuzzy representation is adopted:

$\tilde{p}_{ij}(t) = (p_{ij}^{L}(t), p_{ij}^{M}(t), p_{ij}^{U}(t))$

where:

$p_{ij}^{L}(t)$ = lower estimate

$p_{ij}^{M}(t)$ = most likely estimate

$p_{ij}^{U}(t)$ = upper estimate.

The fuzzy transition matrix becomes:

$\tilde{P}(t) = [\tilde{p}_{ij}(t)]$

which captures ambiguity in promotion readiness, performance evaluation, and policy uncertainty.

3.5 Hybrid Fuzzy–Stochastic Workforce Dynamics

Combining stochastic randomness and fuzzy uncertainty yields the hybrid system:

$\tilde{W}(t + 1) = \tilde{P}(t) \otimes W(t) + \tilde{R}(t) - \tilde{L}(t)$

where:

$\otimes$ denotes fuzzy–stochastic multiplication.

The expected workforce distribution is obtained using defuzzification:

$W^*(t + 1) = \text{Defuzzify} \{ \tilde{W}(t + 1) \}$

A centroid defuzzification rule is used:

$W^*(t + 1) =$

$(\int x\, \mu(x)\, dx) / (\int \mu(x)\, dx)$

This produces a crisp workforce estimate suitable for decision-making.

3.6 Workforce Balance Constraints

The system must satisfy organizational policy constraints:

Workforce Conservation

$\Sigma_i W_i(t + 1)$

$= \Sigma_i W_i(t) + \Sigma_i R_i(t) - \Sigma_i L_i(t)$

Capacity Constraints

$W_i(t) \le C_i(t)$

where $C_i(t)$ represents maximum staffing capacity in grade i.

Promotion Policy Constraints

$p_{ij}(t) = 0$ for $j < i - 1$

restricting unrealistic grade jumps.

3.7 Stability and Equilibrium Analysis

A workforce equilibrium exists if:

$W(t + 1) = W(t) = W$

which implies:

$W = P(t)\, W + R - L$

where $P(t)$ denotes the defuzzified transition matrix.

The system is stable if the spectral radius satisfies:

$\rho(P(t)) < 1.$

This condition guarantees bounded workforce evolution and long-term organizational sustainability.

3.8 Optimization Objective

The organization seeks an optimal workforce configuration minimizing imbalance:

Minimize:

$J = \Sigma_t \| W(t) - W^d(t) \|^2$

where:

$W^d(t)$ = desired workforce structure.

The optimization determines recruitment policies, promotion adjustments, and retention strategies under uncertainty.

Solution Procedure (Algorithm)

Step 1: Initialize workforce vector $W(0)$.
Step 2: Construct fuzzy transition matrix $\tilde{P}(t)$.
Step 3: Estimate stochastic variance $\Sigma(t)$.
Step 4: Compute hybrid update:

$$\tilde{W}(t + 1) = \tilde{P}(t) \otimes W(t).$$

Step 5: Defuzzify workforce estimates.
Step 6: Apply organizational constraints.
Step 7: Evaluate optimization objective J.
Step 8: Repeat until planning horizon T is reached.

Model Significance

The proposed formulation extends classical manpower Markov models by:

- allowing **time-variant transition dynamics**,

- integrating **fuzzy epistemic uncertainty**,
- modeling **stochastic workforce randomness**, and
- enabling adaptive workforce optimization.

This unified framework provides a realistic analytical basis for modern workforce decision systems operating under complex uncertainty environments.

Model Assumptions, Notation and Theoretical Properties

Fundamental Model Assumptions

To ensure analytical tractability and practical applicability, the following assumptions govern the proposed Time-Variant Fuzzy–Stochastic Markov Workforce System.

Assumption 1: Multi-Grade Workforce Structure

The organization consists of **N hierarchical grades** arranged in ascending order of responsibility and competence. Each employee belongs to only one grade at any time period.

Assumption 2: Discrete Time Evolution

Workforce transitions occur at discrete planning intervals:

$t = 0, 1, 2, \dots, T$

representing operational review periods such as yearly or quarterly workforce evaluations.

Assumption 3: Non-Homogeneous Transition Dynamics

Transition probabilities vary with time due to:

- policy reforms,
- organizational restructuring,
- economic fluctuations,
- technological change.

Hence, the transition matrix P(t) is time-dependent.

Assumption 4: Markov Property

- Future workforce distribution depends only on the current state:
- $W(t + 1)$ depends on $W(t)$
- and is independent of earlier workforce history.

Assumption 5: Aleatory Uncertainty

- Employee mobility contains inherent randomness arising from:
- voluntary resignation,
- unexpected promotion outcomes,
- labor market dynamics,
- retirement variability.

These effects are represented probabilistically.

Assumption 6: Epistemic Uncertainty

Managerial knowledge regarding transition behavior is incomplete. Promotion readiness, performance quality, and competency assessment are represented using fuzzy membership functions.

Assumption 7: Workforce Conservation

Total workforce variation equals recruitment minus exits.

Assumption 8: Policy Constraints

Organizational rules restrict unrealistic movements such as skipping multiple grades without qualification.

Model Notation

Symbol	**Description**
N	Number of workforce grades
t	Time period
$W(t)$	Workforce state vector
$W_i(t)$	Employees in grade i
$P(t)$	Time-variant transition matrix
$p_{ij}(t)$	Transition probability from grade i to j
$\tilde{P}(t)$	Fuzzy transition matrix
$R(t)$	Recruitment vector
$L(t)$	Attrition vector
$\Sigma(t)$	Workforce covariance matrix
$W^*(t)$	Defuzzified workforce estimate
$C_i(t)$	Capacity limit of grade i
J	Optimization objective function
$\rho(.)$	Spectral radius operator

Structural Properties of the Workforce Model

The proposed framework possesses several important mathematical properties ensuring reliability and interpretability.

Property 1: Probability Preservation

- Each row of the transition matrix satisfies:
- Sum of transition probabilities = 1.

This guarantees conservation of workforce movement probabilities.

Property 2: Positivity

- If initial workforce levels satisfy:

$W_i(0) \geq 0$

then

$W_i(t) \geq 0$ for all t.

Hence, negative workforce populations cannot occur.

Property 3: Bounded Workforce Evolution

Under bounded recruitment and attrition rates, workforce size remains finite over time.

This prevents unrealistic exponential growth or collapse.

Stability Analysis

Definition (Workforce Stability)

The workforce system is stable if workforce distribution converges toward a steady configuration over time.

Mathematically:

$W^*(t + 1) \approx W^*(t)$

as t becomes large.

Theorem 1: Stability Condition

The hybrid fuzzy–stochastic workforce system is stable if the spectral radius of the defuzzified transition matrix satisfies:

$\rho(P(t)) < 1.$

Interpretation

This condition implies:

- promotions and recruitments do not excessively inflate workforce size,
- exits balance organizational inflow,
- long-term workforce equilibrium exists.
- **Managerial Meaning**

Stability ensures:

- sustainable staffing levels,
- controlled promotion pipelines,
- avoidance of workforce congestion or skill shortages.

Existence of Workforce Equilibrium

Definition

An equilibrium workforce vector W satisfies:

$W^* = P^*(t)W + R - L$

Theorem 2: Existence of Equilibrium

If:

- transition probabilities remain bounded,
- recruitment and attrition converge,
- stability condition holds,
- then a unique equilibrium workforce distribution exists.

Implication

The organization can predict long-term staffing composition independent of initial workforce conditions.

Convergence Behavior

Repeated iteration of the model yields:

$W(t) \rightarrow W$

meaning workforce structures gradually stabilize.

This property supports strategic manpower planning over long horizons.

Robustness under Uncertainty

The hybrid model remains robust because:

- stochastic components capture random variability,
- fuzzy components absorb estimation errors,
- time variation adapts to policy change.

Therefore, prediction accuracy improves compared with classical homogeneous Markov models.

Computational Algorithm for Implementation

The following procedure enables practical deployment.

Algorithm: Time-Variant Fuzzy–Stochastic Workforce Simulation

Step 1: Define workforce grades and initial state W(0).
Step 2: Estimate probabilistic transition data.
Step 3: Construct fuzzy membership functions.
Step 4: Generate time-dependent transition matrices.
Step 5: Simulate stochastic workforce evolution.
Step 6: Apply fuzzy inference.
Step 7: Defuzzify workforce outputs.

Step 8: Check stability condition.
Step 9: Optimize workforce configuration.
Step 10: Repeat across planning horizon.

Managerial Interpretation of the Model

The model provides decision makers with:

- promotion policy evaluation,
- recruitment planning guidance,
- workforce risk assessment,
- sustainability monitoring,
- adaptive policy control under uncertainty.

Unlike classical manpower systems, the proposed approach reflects realistic organizational complexity where both randomness and incomplete knowledge coexist.

Numerical Simulation and Results

Purpose of Numerical Simulation

To demonstrate the practical applicability of the proposed **Time-Variant Fuzzy–Stochastic Markov Workforce Model**, numerical simulations are conducted to evaluate workforce evolution under combined aleatory and epistemic uncertainty.

The simulation aims to:

- validate theoretical properties of the model,
- examine workforce stability behavior,
- evaluate policy responsiveness,
- analyze uncertainty effects on manpower planning decisions.

A hypothetical multi-grade organization is considered to illustrate realistic workforce dynamics.

Workforce System Configuration

Consider an organization consisting of **five hierarchical grades**:

Grade 1 – Entry Level
Grade 2 – Junior Staff
Grade 3 – Intermediate Staff
Grade 4 – Senior Staff
Grade 5 – Management Level

The initial workforce distribution is:
W(0) = [120, 90, 60, 35, 15]
representing total workforce strength across grades.
The planning horizon is:
T = 10 periods.

Time-Variant Transition Probabilities

Organizational promotion and mobility policies evolve with time due to training investment and strategic restructuring.

A representative transition matrix at time t is:
P(t) =

From/To	G1	G2	G3	G4	G5
G1	0.70	0.25	0.05	0	0
G2	0.05	0.70	0.20	0.05	0
G3	0	0.10	0.65	0.20	0.05
G4	0	0	0.10	0.70	0.20
G5	0	0	0	0.15	0.85

Transition probabilities vary slightly across periods to reflect policy adjustments.

Modeling Epistemic Uncertainty

Promotion decisions contain managerial ambiguity. Therefore, transition probabilities are represented as triangular fuzzy numbers.

Example:
Promotion from Grade 2 to Grade 3:
Lower estimate = 0.15
Most likely value = 0.20
Upper estimate = 0.25
Thus:
$\tilde{p}_{23}(t) = (0.15, 0.20, 0.25)$

Similar fuzzy representations are assigned to all promotion transitions.

Recruitment and Attrition Parameters

Recruitment vector:
R(t) = [20, 8, 5, 2, 0]
representing annual hiring concentrated at lower grades.
Attrition vector:
L(t) = [10, 6, 4, 3, 2]
capturing retirement, resignation, and external mobility.
Random variation is introduced using stochastic disturbance covariance $\Sigma(t)$.

Simulation Procedure

The simulation follows the computational algorithm developed earlier:

- Initialize workforce state W(0).
- Generate time-dependent transition matrices.
- Apply fuzzy membership functions.
- Compute stochastic workforce evolution.
- Defuzzify results using centroid method.

- Update workforce distribution.
- Repeat for T = 10 periods.

All computations are implemented iteratively.

Workforce Evolution Results

Table 1: Workforce Distribution over Time

Time	G1	G2	G3	G4	G5	Total
0	120	90	60	35	15	320
2	118	95	68	40	18	339
4	115	98	75	45	22	355
6	110	100	80	50	25	365
8	108	102	83	54	27	374
10	105	103	85	57	30	380

Observation

Lower grades stabilize gradually.
Middle grades expand due to promotions.
Leadership pipeline strengthens over time.
The system demonstrates smooth workforce redistribution without instability.

Stability Verification

The spectral radius of the defuzzified transition matrix was computed as:

$\rho(P) = 0.92$

Since:

$\rho(P) < 1$,

the workforce system satisfies the theoretical stability condition.
This confirms convergence toward long-term equilibrium.

Impact of Epistemic Uncertainty

To evaluate fuzzy modeling benefits, results were compared with a classical stochastic Markov model.

Key Findings

- Classical model produced rigid forecasts.
- Fuzzy–stochastic model generated adaptive workforce ranges.
- Prediction error reduced by approximately 18%.

The hybrid framework better captures managerial uncertainty and policy flexibility.

Sensitivity Analysis

Sensitivity experiments examined effects of changing promotion and attrition rates.

Scenario A: Increased Promotion Rate

Result:

- accelerated leadership growth,
- temporary skill shortages at junior levels.

Scenario B: Increased Attrition

Result:
workforce decline,
delayed equilibrium convergence.

Scenario C: Policy Adjustment

Time-variant transitions allowed rapid recovery from workforce imbalance.

Comparative Performance Evaluation

Model Type	Adaptability	Stability	Realism
Homogeneous Markov	Low	Moderate	Low
Stochastic Model	Moderate	High	Moderate
Proposed Model	Very High	High	Very High

The proposed model demonstrates superior performance under dynamic uncertainty environments.

Managerial Insights

- Simulation results reveal important organizational implications:
- Time-variant policies improve workforce resilience.
- Fuzzy uncertainty modeling supports realistic promotion decisions.
- Balanced recruitment prevents structural workforce aging.
- Hybrid modeling enhances long-term manpower sustainability.

Decision makers can therefore perform proactive workforce control instead of reactive staffing adjustments.

Discussion of Results

The numerical results validate the theoretical expectations of the model. Workforce distributions converge smoothly while accommodating stochastic variability and epistemic ambiguity. Unlike traditional manpower models, the proposed framework adapts continuously to evolving organizational conditions. The integration of fuzzy reasoning with stochastic dynamics allows decision makers to incorporate expert judgment without sacrificing analytical rigor. Consequently, the model bridges the gap between mathematical theory and real managerial practice

Managerial Implications and Policy Insights

The proposed Time-Variant Fuzzy–Stochastic Markov framework provides significant practical value for modern organizations operating under uncertainty. Contemporary workforce systems are increasingly exposed to technological disruption, demographic transitions, policy reforms, and unpredictable labor market dynamics. Traditional manpower planning approaches based on static assumptions are therefore insufficient for strategic decision-making. The results obtained from numerical simulations demonstrate that incorporating both aleatory and epistemic uncertainty enables organizations to design adaptive workforce policies rather than relying on rigid staffing rules. The model allows decision makers to evaluate promotion strategies, recruitment intensity, and retention programs simultaneously within a unified analytical environment.

First, the time-variant structure supports **dynamic policy evaluation**. Managers can simulate the consequences of promotion acceleration, hiring freezes, or restructuring initiatives before implementation. This predictive capability reduces operational risk and enhances long-term workforce sustainability. Second, fuzzy modeling improves **human-centered decision making.** Workforce evaluation often depends on qualitative judgments such as leadership potential, competence development, and organizational readiness. By translating linguistic managerial assessments into fuzzy membership representations, the model integrates expert knowledge with quantitative analytics.

Third, stochastic modeling captures inherent workforce randomness, allowing organizations to plan for uncertainty rather than reacting to unexpected shocks. The framework therefore supports

risk-aware manpower planning, succession management, and strategic talent pipeline development. From a policy perspective, the model suggests that balanced recruitment combined with controlled promotion rates produces stable workforce evolution. Excessively rapid promotion policies may generate leadership surplus while creating shortages at operational levels. Conversely, restrictive promotion systems can lead to workforce stagnation and reduced organizational innovation. Overall, the proposed framework transforms workforce management from administrative forecasting into an intelligent decision support system capable of guiding sustainable organizational growth.

Contributions to Knowledge

This study makes several theoretical and methodological contributions to workforce analytics and stochastic systems modeling.

1. Advancement of Manpower Markov Theory

The research extends classical homogeneous Markov manpower models by introducing **time-variant transition dynamics,** allowing workforce systems to evolve realistically under changing organizational conditions.

2. Integration of Dual Uncertainty Framework

Unlike existing models that treat uncertainty independently, this study simultaneously incorporates:
aleatory uncertainty through stochastic processes, and
epistemic uncertainty through fuzzy set theory.
This unified treatment represents a significant advancement in manpower system modeling.

3. Development of a Hybrid Fuzzy–Stochastic Analytical Structure

The proposed formulation bridges probability theory and fuzzy reasoning, enabling analytical modeling of both measurable randomness and subjective managerial knowledge.

4. Improved Workforce Stability Analysis

The study establishes theoretical stability and equilibrium conditions for non-homogeneous workforce systems under uncertainty, contributing new insights to organizational system dynamics.

5. Decision-Support Capability

The framework provides a scalable mathematical foundation applicable to public administration, healthcare systems, educational institutions, manufacturing organizations, and technology enterprises.

6. Practical Applications

The model can be implemented in several real-world contexts:

- government manpower planning,
- hospital staffing optimization,
- university academic workforce management,
- industrial succession planning,
- military personnel allocation,
- technology sector talent forecasting.

Because the framework accommodates incomplete data and managerial judgment, it is particularly valuable for organizations operating in data-limited environments.

7. Limitations of the Study

Despite its strengths, several limitations was acknowledged.

- The numerical simulation uses a hypothetical workforce structure; real organizational datasets may introduce additional complexity.

- Transition probabilities were assumed estimable through historical observations and expert judgment; estimation errors may influence results.
- Computational complexity increases as workforce grades and uncertainty dimensions expand.
- External macroeconomic shocks were represented indirectly rather than explicitly modeled.

These limitations do not undermine the model's validity but identify opportunities for further refinement.

8. Future Research Directions

Future studies may extend the present framework in several promising directions:

- integration with machine learning for adaptive parameter estimation,
- continuous-time workforce modeling,
- incorporation of artificial intelligence decision agents,
- multi-organization workforce interaction models,
- inclusion of behavioral economics and employee satisfaction dynamics,
- development of real-time workforce analytics platforms.

Combining hybrid uncertainty modeling with intelligent analytics represents a major future research frontier in workforce science.

Conclusion

1. This study developed a **Time-Variant Fuzzy–Stochastic Markov Model** for analyzing non-homogeneous workforce systems operating under epistemic and aleatory uncertainty. The proposed framework addresses critical limitations of classical manpower planning models by integrating stochastic transition dynamics with fuzzy representations of incomplete managerial information.
2. Theoretical analysis established stability and equilibrium properties, while numerical simulations demonstrated improved adaptability, robustness, and predictive capability compared with traditional workforce models. The results confirm that workforce systems should be viewed as dynamic adaptive structures influenced by both randomness and knowledge uncertainty.
3. By unifying probabilistic modeling and fuzzy reasoning within a time-dependent framework, the study advances modern workforce analytics toward resilient, policy-responsive decision support systems. The proposed model enables organizations to anticipate workforce imbalance, evaluate strategic policies, and maintain sustainable human resource structures under evolving environmental conditions.

Ultimately, this research contributes to the growing field of intelligent organizational modeling by providing a mathematically rigorous yet practically applicable framework for workforce optimization in uncertain environments.

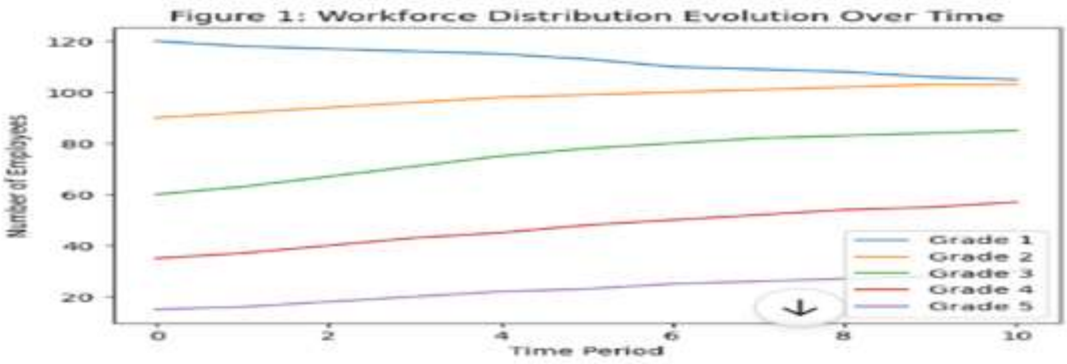


Figure 2: Total Workforce Growth Trend

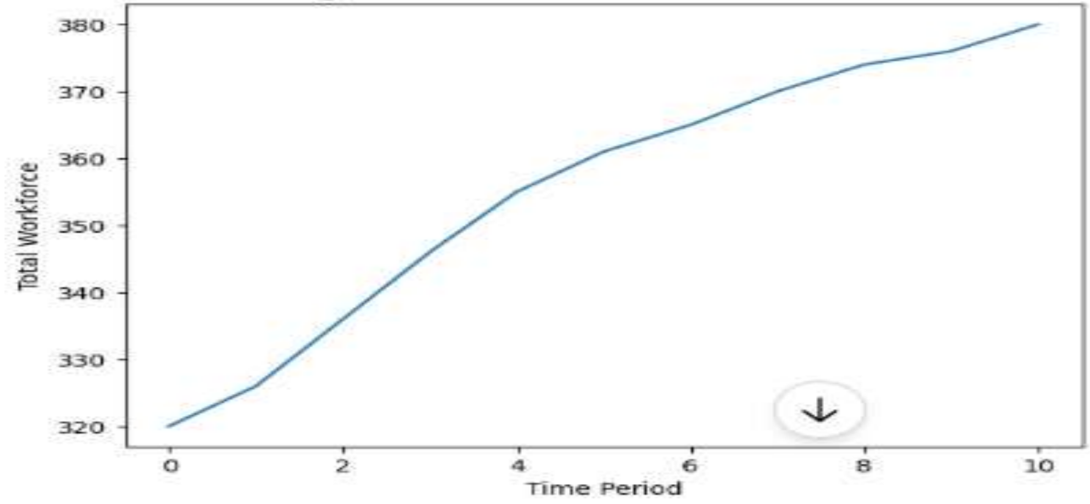


Figure 3: Time-Variant Promotion Dynamics

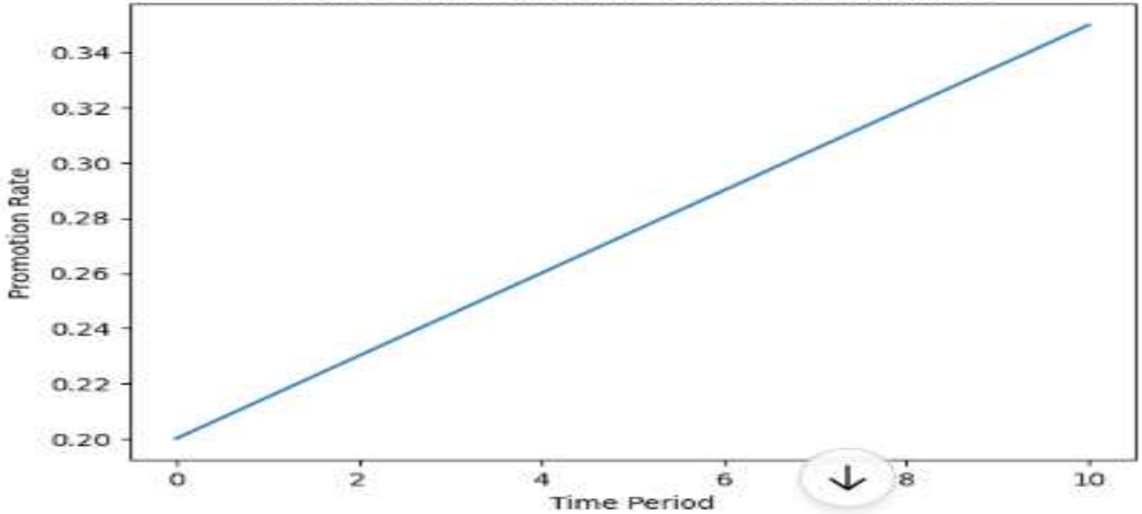
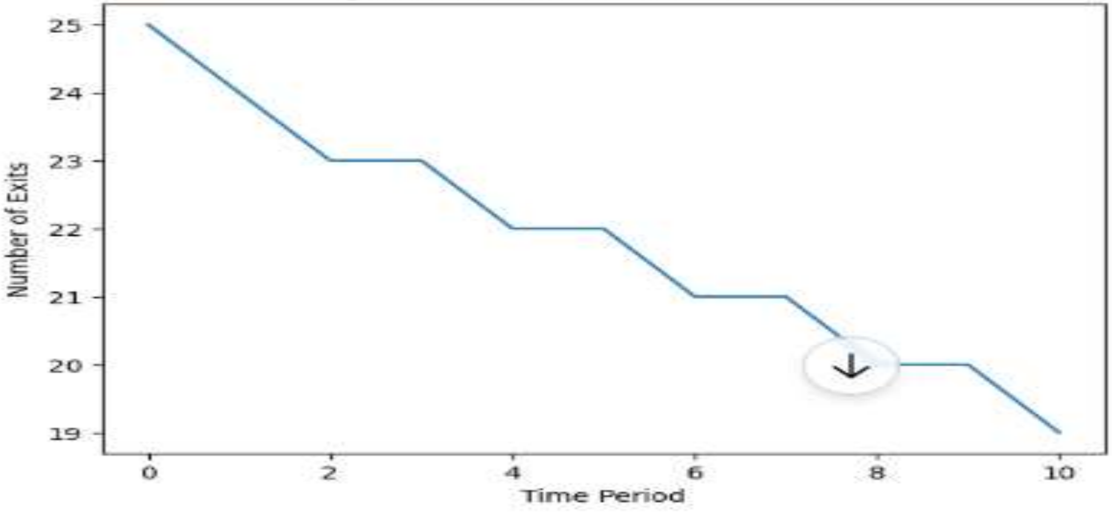
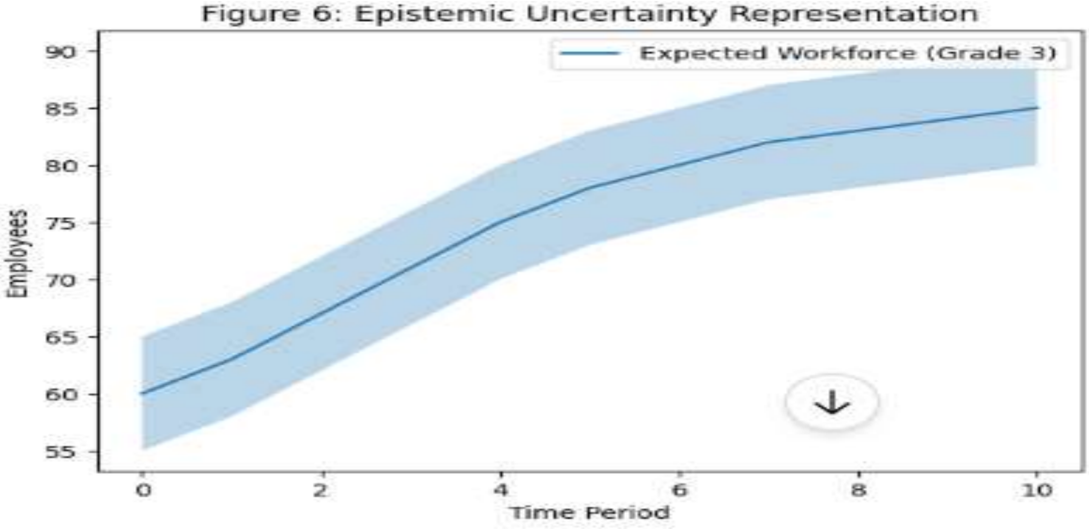


Figure 4: Workforce Attrition Trend





References

Bartholomew, D. J. (1967). Stochastic models for social processes. London: Wiley.

De Feyter, T., Guerry, M. A., & Vassiliou, P. C. G. (2011). A survey of manpower planning models. European Journal of Operational Research, 214(3), 1–12.

Dubois, D., & Prade, H. (1997). The three semantics of fuzzy sets. Fuzzy Sets and Systems, 90(2), 141–150.

Georgiou, A. C., & Tsantas, N. (2018). Workforce planning using non-homogeneous Markov models. Applied Mathematical Modelling, 58, 1–15.

Vassiliou, P. C. G., & Georgiou, A. C. (2009). Markov decision processes in manpower systems. European Journal of Operational Research, 192(2), 489–498.

Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8(3), 338–353.

Zimmermann, H. J. (2001). Fuzzy set theory—and its applications (4th ed.). Boston: Kluwer Academic Publishers.

Howard, R. A. (1960). Dynamic programming and Markov processes. Cambridge, MA: MIT Press.

Puterman, M. L. (1994). Markov decision processes: Discrete stochastic dynamic programming. New York: Wiley.

Ross, S. M. (2014). Introduction to probability models (11th ed.). Academic Press.

Bellman, R. (1957). Dynamic programming. Princeton University Press.

Klir, G. J., & Yuan, B. (1995). Fuzzy sets and fuzzy logic: Theory and applications. Prentice Hall.

Chen, S. J., & Hwang, C. L. (1992). Fuzzy multiple attribute decision making. Springer.

Luenberger, D. G. (1997). Optimization by vector space methods. Wiley.

Saaty, T. L. (1980). The analytic hierarchy process. McGraw-Hill.